Article

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ASSESSING WAITING TIME MANAGEMENT STRATEGIES AND PATIENT SATISFACTION AT MATER MISCERICORDIAE AFIKPO

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Abstract
Long queues of accident and emergency units waiting to be attended to are a global healthcare concern affecting most hospitals (general and specialists), especially in Nigeria. The problem of excessive wait time in the Accident and Emergency Department (AED) is a barrier to receiving access to assessment and treatment for patients seeking care in the Ebonyi State. This research sought to understand the success factors that help implement and sustain wait time management strategies (WTMS) and ensure reduction of wait times in Mater Miscericordiae hospital Accident and Emergency Departments. A total of 208 patients were clacked in and out with to ascertain their waiting time. The result is a delay in treatment and suboptimal outcomes for newly referred patients. We recommend that the management of the hospital should increase the number of servers at the general out-patient department to three or four so as to reduce the time patients spend on queue before services thereby, minimizing the cost incurred from waiting and also increases patient's satisfaction with the services rendered.

Keywords: Accident and Emergency Department (AED), waiting time, customer satisfaction, service

Introduction
Accessing and utilization of medical services is influenced by a number of factors, one of which is the length of time patients must wait to be attended to. Long wait times are seen by patients as a barrier to receiving treatments, and keeping patients waiting longer than necessary can be stressful for the patient, caregiver, and clinician. Within the topic of operations management, queuing theory is the theoretical study of standing in a line. Nearly all operations in the social and manufacturing spheres, according to Davis et al. (1996), may be thought of as gates. Each unit of natural or human being arriving at the gate needs to provide a particular amount of service time before it may pass on to the next unit of human being. The reciprocal of the service time is, of course, the rate capacity of this gate. Hospitals operate in an increasingly cutthroat economy, therefore in order to retain the same number of patients, if not more, they must prioritize patient happiness. Barua (2017) reported that a Canadian Fraser Institute report found that the average wait time for first appointments was 10.2 weeks when the clinically reasonable limit, according to specialists, is 7.2 weeks. In some cases, patients were waiting for more than 21 weeks. Despite

prior efforts to reduce wait times across the country, adverse medical consequences continued to occur as a result of prolonged wait times (Day, 2013). These include patient safety problems such as poorer medical outcomes and an increased risk of adverse events and economically, an increased cost burden (Barua, 2017). A system's thinking approach allows managers to assess the interrelation of numerous factors (Marchal et al., 2012). According to Gravelle and Siciliani (2008), rationing the demand for health services by waiting list is inefficient and creates a deadweight loss to the society. Many studies have shown that patients' satisfaction has many benefits. Utilization of health services is better when the quality of care is perceived to be good (Onyeneho, Amazigo, Njepuome, Nwaogu, and Okeibunor, 2016) while low quality of care is a barrier to utilization (Bakeera, Wamala, Galea, State, Peterson and Pariyo, 2009). Excessive wait time caused by overcrowding creates a snowball of negative effects including poor patient outcomes, prolonged pain and suffering, patient dissatisfaction and decreased physician productivity and overall frustration by the healthcare staff (Derlet & Richards, 2000). Customers' preferred strategy will be comprised of the actions yielding the shorter expected waiting time for any given system state.

Organisational Context

Mater Misericordiae Hospital is a state-owned hospital managed by the catholic church offering general outpatients, medical surgery and general treatment. Mater Misericordiae Hospital is a Private hospital, located at Ohaisu Afikpo B, Afikpo North Local Government, Ebonyi State. Mater Miscericordiae hospital has capacity of 450 beds and over 1700 employees comprising tenured and non-tenured staff, the hospital is one of Ebonyi state's largest urban hospitals and registered as Primary Health Care Centre. Medical Services include Cardiology, Gastroenterology, Hematology, Geriatrics, Neurology, Pulmonology, Infectious Diseases, Family Medicine, General Surgery, Radiology, and Neonatology, Obstetrics, Gynecology and Special Clinical services such as Antenatal Care (ANC), Immunization, HIV/ AIDS Services, Tuberculosis, Non Communicable Diseases, Family Planning, Communicable Diseases, Hepatitis, Accidents and Emergency, Nutrition, Health Education and Community Mobilization, Maternal and newborn care, Scanning.

LITERATURE REVIEW.

Queuing Theory

A queue in general, is formed at any place when a customer (human beings or physical entities) that requires service is made to wait due to the feet that the number of customers exceeds the number of service facilities or when service facilities do not work efficiently and take more time than prescribed to serve a customer (Sharma, Kumar and Sharma, 2013). It involves customer waiting in line to receive services in any service system. Queuing theory in the other hand, is the study of waiting in all various guises (Hilier & Lieberman 2001). It is a theory that uses queuing models to represent the various types of queuing system that arises in practice. A queuing theory can be applied to a variety of situations where it is not possible to accurately predict arrival date/time of customers and service rate/time of service facilities. Particularly, it can be used to ascertain the level of service (either the service rate or the number of service facilities) that balances the cost of offering the service. That i.e., it enables an appropriate model that balances between the cost of services and the amount of waiting (Onwujekwe, Etiaba & Oche, 2015; Sharma, Kumar and Sharma, 2013; Hilier & Liebermen, 2001). Therefore, queuing theory is a body of knowledge that incorporates queuing process, system and models.

In practically every part of the world, queue is one of the key challenges confronting hospitals. In hospitals in Nigeria, patients must wait a long time even for a minor procedure. The theory of queues has various uses, and the majority of them have been well explored in the literature on probability, operations research, and management science. Machine repair, tool booths, inventory control, ship loading and unloading, patient scheduling in hospitals, use in computer fields, and other applications are a few examples of areas of applicability. However, the most significant feature of queuing theory is its widespread application in hospitals and healthcare settings to speed up patient service while also reducing the perceived length of time patients must wait. In this phase of the research work, empirical studies on the impact of waiting times on patients' happiness are reviewed, along with the implications for how Nigerian hospitals can handle queue management.

The work of Cruz and T. Van Woensel (2014), solved the issue of Multiple-Server Finite Capacity Queuing Model by effective process time distribution. Anish Amin, Piyush Mehta, Abhilekh Sahay, Pranesh Kumar and Arun Kumar (2014), applied a new method on model (Q/Q/ n) to reduce waiting time of customer at Toll system. M. Reni Sagayraj, R. Raguvaran and M. Daisy (2015), worked on Multiple-Server Queuing Model to minimize the waiting time of customers in the fuel station. Hajnal Vass and Zsuzsanna K. Szabo (2015), suggested a model (Q/Q/3) to analyze the patient flow in the Emergency Department. Rashmita Sharma (2015), worked on Finite Capacity Multiple-Server Queuing Model where customers are arriving in bulk. Nilesh Sheth and Prashant Makwana (2016). Abdul Wahab, Nawusu Yakubu and Ussiph Najim (2014), worked on the model (Q/Q/ n) of bank ATM. Tariq Ahmad Koka and V. H. Badshah (2016), gives the Quick Pass model (Q/Q/ n) in the railway ticket counter. Tariq Ahmed Koka, V. H. Badshah and Riyaz Ahmad Shah (2017), getting better results for Multiple-Server Queuing model.

Long hospital waits have always been a sensitive political issue in many OECD countries, especially in those nations where the health care is publicly funded (Siciliani & Hurst, 2005). It causes large dissatisfaction among patients and has generally been considered as a problem resulted by inefficiency in the healthcare system (Bhattacharya, Hyde, & Tu, 2014). Boyce, Kraft, Svenonius & Borko (1991) states that queuing theory enables mathematical analysis of several related process, including arriving at the queue, waiting in the queue, and being served at the front of the queue. The theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service, and the probability of encountering the system in certain states, such as empty, full, having an available server or having to wait a certain time to be served. Nosek and Wilson (2001) reviewed the use of queuing theory in pharmacy applications with particular attention to improving customer satisfaction. Customer satisfaction is improved by predicting and reducing waiting times and adjusting staffing.

Furthermore, advances in technology such as computers, internet etc., have provided firms with the ability to provide faster services. For these reasons’ physicians, bank administrators, and managers are continuously finding means to deliver faster services believing that the waiting will affect after service evaluation negatively. Also, understanding the inefficiencies in the bank and improving them is crucial for making banking policy and budgeting decisions. Wilson and Nguyen (2004).

Kembe, Onah & Iorkegh (2006) studied the queuing characteristics at the Riverside Specialist Clinic of the Federal Medical Centre Makurdi was analyzed using a multi-server queueing model and the waiting and service costs determined with a view to determining the optimal service level. Adeleke, Ogunwale & Halid (2009) in their study obtained the traffic intensity to be $\rho = 0.8444$ is the probability of patients given on arrival which clearly indicate a higher possibility of patients waiting for treatment since the doctor is busy rendering service to a patient that has earlier arrived. He added that excessive waste of time in the hospital may lead to patients' health complications and in some cases eventual death which may be avoided. As a result, it is recommended that more number of doctors should be deployed to this centres so as to convert a single-channel queuing unit to multi-channel queuing units. This will help in offering service on arrival. Aksin-Karaesmen, Baris, Emadi & Chei-Lin (2011) modeled callers’ abandonment decision as an optimal stopping problem in a call center context and found heterogeneity in callers waiting behavior. The study also looked at customers’ heterogeneity in-waiting sensitivity and related this sensitivity to customers’ price sensitivity. They found that the association between Price and waiting sensitivity as important managerial implications.

Healthcare systems operation such as patient scheduling, resource scheduling, queue length, Limited Queue Discipline (LQD), blocking, healthcare systems design and analysis are crucial parts of study in healthcare management. The effective usage of resources, high quality of service and less queues are the main aims that needed to evaluated for the successes of policlinics and hospitals (Lakshmi and Sivakumar, 2013).

The Malaysian health ministry has set the patient's waiting time for 60 minutes. This guideline is as noted in the management of government hospitality and customer charter. However, forgot the outpatient hospital departments, Bahadori, Ravangard, Raadabadi, Mosavi, Fesharaki & Mehrabian (2014) suggested the best waiting time is between 8-10 minutes. This situation clearly demonstrates the optimum waiting time difference between the Malaysian Health Ministry customer charters with the proposed standard. Although many patients have chosen a private health center rather than a government health clinic for treatment because of the quality of service and doctor's expertise, government clinics are still plagued with long waiting times. Long waiting times in health centers can lead to increase the severity of disease and cause socioeconomic costs.

In a research to examine the curing process and its application to customer's service delivery in fidelity bank Plc. Bakari, Chamalwa & Baba (2014) obtained from the study the value of the traffic intensity, otherwise known as the utilization factor to be less than one (i.e. $\rho<1$) stating that the system operates under steady-state condition. Does the value of the traffic intensity which is the probability that the system is busy implies that 95% of the time Considered during data collection the system was busy as against 4% at the time the data is been collected. This indicates high utilization of the system.

Hajnal and Zsuzsanna (2015) in their study of the application of queuing model in to patient flow in Emergency Department (ED) overcrowding represent a common characteristic that may affect the quality and access to healthcare. For each ED it is a challenge to improve the patient's satisfaction. Out of 2195 questionnaires, the satisfaction rate is 84.63% and the most frequent complaints are about long waiting time, small waiting room and insufficient personnel. Hajnal and Zsuzsanna concluded in their work that the M/M/3 queue model should be used to characterize the patient flow in the ED. This model can be invaluable in providing decision support in complex environment as the ED. Agyei *et al.* (2015) in their study attempts to find the trade-off between minimizing the total economic cost (waiting cost and service cost) and the provision of a satisfactory and reasonably shortest possible time of service to customers, in order to assist management of firms in deciding the optimal number of servers needed.

Kyoung *et al.* (2017) in their study of the application of queuing theory to analyze changes in outpatient's waiting times before and after the introduction of Electronic Medical Record (EMR) system. The two fundamental parameters for queuing analysis, arrival rate (λ) and service rate (μ), is there from digital data to apply queuing theory to the analysis of outpatient waiting times. The result obtained from using queuing theory to calculate the waiting time, they used the M/M/1 model in the methodology to analyze changes in the outpatients' waiting times before and after the introduction of EMR. Then they concluded that it is possible to analyze waiting times while minimizing input errors and limitations influencing consultations procedures if they use digital data and apply the queuing Theory. Their results verified that the introduction of electronic medical record (EMR) contribute to the improvement of patient services by decreasing outpatient waiting time or by increasing efficiency. Ariff*et al.* (2018) in their study, after discussing the problems in cured and made possible recommendation concluded in their study that queuing simulation and scheduling are combinations of powerful management tool which often applied in the medical sector. Proper implementation of these powerful management tools may lead to an effective and efficient management system.

Aziati and Salsabilah (2018) states the main objective of this study is to determine waiting time of patient's floor which can be used to improve the operating performance and also improving the quality of the service provided to the patients. The study further established the green theory and modeling is an effective tool that can be used to make decision on staffing needs for optimal performance with regards to queuing challenges in hospital. The study recommends the management should invest in using the new technologies or systems that can be implemented at the public health clinic, especially in the outpatient department. Only a simple flow can be developed. Hajar *et al.* (2018) in their study of reviewing the queuing problem in healthcare. A comprehensive summary, simulation and scheduling theory used in waiting time, appointment system and patient flow are summarized in their study in different departments in the healthcare system such as the ED, outpatient department and the pharmacy. They discussed that both scheduling and simulation are important to ensure queuing problem circumstances. They concluded that the combination of these powerful management tools which is often applied in many different departments of the hospital and proper implementation of these tools lead to efficient and effective management systems.

Samson *et al.* (2020) in their study, obtained the traffic intensity (ρ) to be 0.96. Since they obtained the value of the traffic intensity, otherwise known as the utilization factor to be less than one (i.e. $\rho<1$), it could be concluded that the system operates under steady-state condition. Thus, the value of the traffic intensity, which is the probability that the system is busy, implies that 95% of the time. Considered during data collection the system was busy as against 4% idle time. This indicates idolization of the system. The conclusion was reached without considering cost models for the system (i.e. the cost of deploying an additional server and cost implication resulting from the firm/organization in ability to provide additional service points). However, the investigator strongly recommends that management should make provision for one additional server to enable her minimize customer waiting time and improve service rates. In research conducted by Jingna *et al.* (2020) states that bed resources at the platform in which most medical and health resources in the hospital play a role and carry the core functions of the health service system. this paper analyses the previous research models of related knowledge of queuing Theory in medical services and summarizes the advantages and disadvantages of the queuing model so that it can be used as a constraint in the subsequent queuing optimization model from the perspective of the hospital and the patient, followed by several indicators such as the average total number of people, the utilization rate of bed resources, etc which makes the patient queue more reasonable. Therefore, as a contribution to the existing literature on queuing theory and service delivery, the study is intended to concentrate on the application of queuing theory in healthcare management especially in Mater Miscericordiae Hospital, Afikpo.

Theories of rationing by waiting list

According to Lindsay and Feigenbaum (1984), the rationing power of hospital waiting lists lies in the "influence of delay on the value of the service delivered". A key assumption in their theory is that the value of a good or service perceived by a demander diminishes when the receipt is delayed. The longer one has to wait to obtain a certain good, the lower the value of that good is perceived by him. In the case of health care, the membership of the hospital waiting list costs nothing in monetary terms, and the queuing itself does not imply that the patient has to wait physically (i.e. no opportunity cost in terms of wasted time). Therefore, the force that brings the demand for and supply of healthcare in equilibrium is the diminishing value of health service perceived by the patient due to waiting, rather than the increasing cost of obtaining such service. Notwithstanding the nature of Mater Miscericordiae Hospital presently, the facility enjoys the Beveridge system (Model) and the Bismarck model of system rationing.

RESEARCH METHODOLOGY

Data source/ limitation

The purpose of this work is to examine the performance characteristics and to determine the productivity and efficiency of using multiple servers by the Management of Mater Miscericordiae Hospital at the Accident and Emergency Department (AED). The system characteristics of interest that will be examined in this project work include; the number of arrivals, service time, the average number of customers in the system, and the average time a patient spends in the system. The result of operating characteristics will be used to evaluate the performance of the service mechanism and to ascertain whether patients are satisfied with the hospital service and also put into consideration the cost and profit of services rendered. Hence, with the aid of observed data, the objectives of this work will be achieved by critically analyzing the real-life data, then constructing a new model system and using statistical analytical tools like exponential, Poisson, and chi-square distribution to study the sequence and reaction to change in the system.

The source of the data used in this research work is a primary source of data collected as patients arrive at the AED. The method applied in collecting data for this project work is through direct observation (primary data) and also introduced a covert observation method during the experiment at the AED. The reason for employing direct observation was to reduce error and obtain reliable data and also, the reason for using covert observation, it occurs when those who are being observed are unaware that someone is observing them. A wrist watch, writing materials were used for the recording of relevant and vital information such as; number of customers, the arrival times of customers, waiting time and service time. The observation was made during the working hours (9:00 am – 1:00 pm) for four days at the AED. The data were collected every Monday for one month.

Method of Data Analysis.

Analysis of the data collected from the hospital is based on:

i. The arrival time of each patient.
ii. The time service commences for each patient and time service ended using the M/M/S model since the system has to do with multiple servers and a single queue.

In formulating the queuing model for this system, the following assumptions was put into consideration;

i. There is single channel queue and a multi-service channel i.e. (M/M/S=2,3,4,5)
ii. The arrival of customers into the system is discrete from Poisson distribution with arrival rate (λ).
iii. The order of arrival and departure are the same and we assume FIFO with infinite queue capacity.
iv. The service channel can only render service of infinite rate exponential distribution with service rate i.e. service time is exponentially distributed.
v. Server will never remain idle if there are one or more jobs in the service node.
vi. Once service is initiated, service of a job will continue until completion.
vii. Arrival in group at the same time (i.e. bulk arrival) is treated as a single arrival.
viii. The waiting area for customers is adequate. The waiting area of the customers in the system is N, which is either limited or unlimited. Hence, the model can be formulated appropriately by using a system for the investment system. Kendall's notation is introduced (A/B/C/D/E).
    A- Which is the arrival distribution or pattern being Poisson
    B- The service time is exponential.
    C- The number of available server S in the system range from two to ten from the assumption above.
    D- This represents the system capacity.
    E- This represent the queue discipline FIFO, which is First in First Out.

Considering the above assumptions and approach, the model formulated is (M/M/S/N/FCFS) by Kendall's notation.

Multiple Servers Queue with M/M/S Model

The M/M/S system is a queuing process having Poisson arrival patterns, S server, with S independent, identically distributed (iid), and Exponential service times (which do not depend on the state of the system). The arrival pattern being stated independent, $\lambda_n = \lambda$ for all n. The service times associated with each server are also independent, but since the number of servers that actually attend to customers (i.e. are not idle) does depend on the service time facility is state-dependent.

If there are number of patients in the queuing system at any point in time, then the following cases may arise:

i. If the number of patients is less than the number of servers, then there will be no queue of patients in the system. Then some of the servers will be idle and the maximum number of patients in the queue will be (c $-$ n).

ii. If the number of patients in the system is more than or equal to the number of servers), then all servers will be busy and the maximum number of patients in the queue will be (n – c).

The parameters for multiple - server model are as follows

λ = Arrival rate
μ= Service rate
c = Number of servers
L_q= Queue Length
N = System capacity
Nq = Average number of customers in the queue.
Ns= Average number of customers in the system.
Tq= the average waiting time in the queue.
Ts = the average waiting time spent in the system.
T_l= Total time lost on queue
$P_n(t)$ = probability that exactly n customers are in queuing system at time t.
$P_w(t)$ = the probability that a customer has to wait at time t
$P_o(t)$ = probability that there are no customers in queuing system in time t.
ρ = Utility factor
ℓ = Traffic intensity

Equations for Multi-channel queuing Model:

Utilization factor: $\rho = \frac{\lambda}{c\mu}$(3.1)

One can choose to use the above equation of $\frac{\lambda}{c\mu} < 1$

If$\frac{\lambda}{c\mu} > 1$, then the waiting line grows larger and larger i.e. becomes infinite if the process runs longer enough. The formulas for the operating characteristics of the multiple-server model are as follow.

cμ> 1 the total number of servers must be able to serve customers faster than they arrive. The probability of zero customer in the system or the probability that the system shall be idle, (Po) is given

$$P_o = \left[\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{c\mu}{c\mu - \lambda}\right]^{-1} \tag{3.2}$$

The probability of n customers (P_n) in the queuing system or the expected number of customers waiting in the queue, is given by $P_n = \frac{\left(\lambda/\mu\right)^n}{c!\times c^{n-c}} P_o\, , for\ n > c$ (3.3)

$$P_n = \frac{\left(\lambda/\mu\right)^n}{c!} P_o\, , for\ n \le c \tag{3.4}$$

When there is one service facility (i.e. C=1), then equation (3.3) and (3.4) can be reduced to,

$$P_n = \left(\lambda/\mu\right)^n \tag{3.5}$$

Probability that a customer has to wait:

$$P_w = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{c\mu}{c\mu - \lambda} P_o \tag{3.6}$$

The average number of customers in the system is (N_s)

$$N_s = \frac{\lambda\mu\left(\lambda/\mu\right)^c}{(c-1)!(c\mu - \lambda)^2} P_o + \frac{\lambda}{\mu}\ \ or\ \ N_s = N_q + \rho \tag{3.7}$$

The average number of customers in the queue is:

$$N_q = \frac{\lambda\mu\left(\lambda/_{\mu}\right)^c}{(c-1)!(c\mu-\lambda)^2} P_o \ or \ N_q = N_s - \rho \qquad (3.8)$$

The average time a customer spends in the queuing system (waiting and being served) is

$$T_q = \frac{N_q}{\lambda} \qquad (3.9)$$

The average waiting time in the system.

$$T_s = \frac{\mu\left(\lambda/_{\mu}\right)^c}{(c-1)!(c\mu-\lambda)^2} P_o + \frac{1}{\mu} \ or \ T_s = \frac{N_s}{\lambda} \qquad (3.10)$$

The average waiting time in the queue

$$T_q = \frac{\mu\left(\lambda/_{\mu}\right)^c}{(c-1)!(c\mu-\lambda)^2} P_o \ or \ T_q = \frac{N_q}{\lambda} \qquad (3.11)$$

Total time lost at waiting per hour

$$T_l = \lambda \times T_q \times 4 \qquad (3.12)$$

Traffic intensity of the queue is given as;

$$\ell = \frac{\lambda}{\mu} \qquad (3.13)$$

These are existing queuing theory models indicating the multi-channel queuing model methods in estimating the parameters.

Data presentation and analysis of patients.

From the data collected, we have that:

Number of patients for four days (N_1, N_2, N_3, N_4)

$N_1 = 65$, $N_2 = 57$, $N_3 = 45$, and $N_4 = 41$

$N = N_1 + N_2 + N_3 + N_4 = 65+57+45+41=208$

Inter-arrival time for 208 patients, T= 595 minutes

Time taken by 208 patients to be served, S= 1044 minutes.

Where T is the total sum of minutes for Inter-arrival time for 208 patients and S is the total sum of minutes for Service time for 208 patients.

Arrival rate: $\lambda = \frac{N}{T} = \frac{208}{595} = 0.3496$

Service rate: $\mu = \frac{N}{S} = \frac{208}{1044} = 0.1992$

Traffic intensity: $\rho = \frac{\lambda}{\mu} = \frac{0.3496}{0.1992} = 1.755 \approx 2 \ patients$

This implies that:

λ = 0.3496 min

μ = 0.1992 min

When there are two servers i.e. C = 2

We now calculate the probability values

(a) Probability that the servers are idle is equation, using (3.2)

$$P_0 = \frac{1}{\frac{(1.755)^2}{2![1-\frac{1.755}{2}]} + 1 + \frac{1.755}{1}} = P_0 = \frac{1}{\frac{3.08}{0.245} + 1 + 1.755} = \frac{1}{15.326} = 0.065$$

(b) Probability of n patients in the system, using (3.3)

For n > c

$$P_n = \frac{(1.755)^n}{2!2^{n-2}}(0.065)$$

For n ≤ c

$$P_n = \frac{(1.755)^n}{2}(0.065)$$

Now we calculate the queue measures

(a) the probability that a customer has to wait at time t, from (3.6)

$$P_w = \frac{1}{2!}\left(\frac{0.3496}{0.1992}\right)^2 \frac{2(0.1992)}{2(0.1992) - 0.3496} 0.065 = 0.8106$$

(b) The expected number of patient in the waiting line, from (3.8)

$$= \frac{(1.755)^3 \times (0.065)}{2 \times 2! \times [1 - \frac{1.755}{2}]^2} = \frac{5.405 \times 0.065}{0.06} = 5.8554 \simeq 6 \; patients$$

(a) The expected number of patient in the system (i.e. waiting plus in service), from (3.7)

= 5.8554 + 2=7.8554 patients

(b) The expected waiting time of patient per minute in the queue is

$$T_q = \frac{N_q}{\lambda} = \frac{6}{0.3496} = 17.16 \approx 17 \; minutes$$

(c) The expected total lost time per hour of patients waiting will be

$$T_1 = \lambda \times T_q \times 4$$

= 0.3496 × 17 × 4 = 23.772 minutes

Now, want to find out if increasing the number of nurses at the GOPD can help to reduce the amount of time spent on the queue and hence minimize the cost incurred by waiting.

Hence we compare the solution with when the number of server is increased (i.e. C=3)

Recall,

Arrival rate: $\lambda = \frac{N}{T} = \frac{208}{595} = 0.3496$

Service rate: $\mu = \frac{N}{S} = \frac{208}{1044} = 0.1992$

Traffic intensity: $\rho = \frac{\lambda}{\mu} = \frac{0.3496}{0.1992} = 1.755$

Now we have,

$$P_0 = \frac{1}{\frac{5.4054}{2.49} + 1 + 1.755 + \frac{(1.755)^2}{2!}} = \frac{1}{6.47} = 0.15$$

(a) Probability that a customer has to wait in time t

$$P_w = \frac{1}{3!}\left(\frac{0.3496}{0.1992}\right)^3 \frac{3(0.1992)}{3(0.1992) - 0.3496} 0.15 = 0.3558$$

(b) The expected number of patient in the queue is

$$N_q = \frac{(1.755)^4 \times (0.15)}{3 \times 3! \times [1 - \frac{1.755}{3}]^2} = \frac{9.4866 \times 0.15}{3.1} = 0.459$$

(c) The expected number in the system is

$$N_S = N_q + \frac{\lambda}{\mu} = 0.459 + 1.755 = 2.214$$

(d) The expected waiting time of patient per hour in the queue is

$$T_q = \frac{N_q}{\lambda} = \frac{0.459}{0.3496} = 1.3129 \text{ minutes}$$

(e) The expected total time lost waiting per hour is

$$T_1 = \lambda \times T_q \times 4$$

$$= 0.3496 \times 1.3129 \times 4 = 1.8356 \text{ minutes}$$

At C = 4 and the other parameter remains the same

$$P_0 = \frac{1}{\frac{(1.755)^4}{4![1 - \frac{1.755}{4}]} + 1 + \frac{1.755}{1!} + \frac{(1.755)^2}{2!} + \frac{(1.755)^3}{3!}}$$

$$P_0 = \frac{1}{\frac{9.4866}{13.47} + 1 + 1.755 + 1.54 + 1.8018} = \frac{1}{6.8011} = 0.147$$

(a) Probability that a customer has to wait in time t

$$P_w = \frac{1}{4!}\left(\frac{0.3496}{0.1992}\right)^4 \frac{4(0.1992)}{4(0.1992) - 0.3496} 0.147 = 0.1035$$

(b) The expected number of patient in the queue is

$$N_q = \frac{(1.755)^5 \times (0.147)}{4 \times 4! \times [1 - \frac{1.755}{4}]^2} = \frac{16.6489 \times 0.147}{30.2401} = 0.0809$$

(c) The expected number of patient in the system is

$$N_s = 0.0809 + 1.755 = 1.8359$$

(d) Then the expected waiting time of patient per hour in the queue is

$$T_q = \frac{N_q}{\lambda} = \frac{0.0809}{0.3496} = 0.2314 \, minutes$$

(e) The expected total time lost waiting per hour is

$$T_1 = \lambda \times T_q \times 4$$

$$= 0.3496 \times 0.2314 \times 4 = 0.3236 \text{ minutes}$$

At C=5 and the other parameters remains the same

$$P_0 = \frac{1}{\frac{(1.755)^5}{5![1 - \frac{1.755}{5}]} + 1 + \frac{1.755}{1!} + \frac{(1.755)^2}{2!} + \frac{(1.755)^3}{3!} + \frac{(1.755)^4}{4!}}$$

$$P_0 = \frac{1}{\frac{16.6489}{77.88} + 1 + 1.755 + 1.54 + 1.8018 + 2.3716} = \frac{1}{8.6822} = 0.1152$$

i. Probability that a customer has to wait in time t

$$P_w = \frac{1}{5!}\left(\frac{0.3496}{0.1992}\right)^5 \frac{5(0.1992)}{5(0.1992) - 0.3496} 0.1152 = 0.0246$$

ii. The expected number of patient in the queue is

$$N_q = \frac{(1.755)^6 \times (0.1152)}{5 \times 5! \times [1 - \frac{1.755}{5}]^2} = \frac{29.2188 \times 0.1152}{252.7206} = 0.0133$$

iii. The expected number of patient in the system is

$$N_s = 0.0133 + 1.755 = 1.7683$$

Then the expected waiting time of patient in the queue is $T_q = \frac{N_q}{\lambda} = \frac{0.0133}{0.3496} = 0.038\ minutes$

i. The expected total time lost waiting in one hour is

$$T_1 = \lambda \times T_q \times 4$$

=0.3496 × 0.038 × 4 =0.0532 minutes.

Table 1: A table of summary showing the result for using different number of servers.

Queue parameter	Two servers	Three servers	Four servers	Five servers
Arrival rate (λ)	0.3496	0.3496	0.3496	0.3496
Service rate (μ)	0.1992	0.1992	0.1992	0.1992
System utilization (ρ)	87.75%	58.5%	43.88%	35.1%
P_o	0.065	0.15	0.147	0.1152
P_w	81.06%	35.58%	10.35%	2.46%
N_q	5.8554	0.459	0.0809	0.0133
N_s	7.8554	2.214	1.8359	1.7683
$T_q(minutes)$	17.16	1.3129	0.2314	0.038
$T_l(minutes)$	23.772	1.8356	0.3236	0.0532

From the table above, the arrival rate and the service rate are 0.3496 and 0.1992 respectively for all servers. The system utilization (ρ), the probability that the servers are idle, probability that the patient has to wait, the number of patients in the queue and the number of patients in the system are 87.75%, 0.065, 81.06%, 5.8554 and 7.8554 respectively for two servers. Likewise, three, four and five servers are shown in the table.

Discussion

Considering the result from the analysis, the system's capacity of 208 patients was studied and the arrival rate was 0.3496 minutes while the service rate is 0.1992 minutes where obtained. The arrival being greater than the service rate implies that patients have to wait, though the patients won't wait for long. The probability that the servers are idle is 0.065 which shows that the servers will be approximately 7 % Idle and 93% Busy.

The expected number in the queue is 5.8554 patients. The expected number in the system is approximately 8 patients. The expected waiting time in the queue is 17.16 minutes and the expected total time lost waiting in one day is 23.722 minutes.

The expected number in the queue is 0.459 patient. The expected number in the system is approximately 2 patients. The expected waiting time in the queue is 1.3129 minutes and the expected total time lost waiting in one day is 1.8356 minutes.

The expected number in the queue is 0.0809 patients. The expected number in the system is approximately 2 patients. The expected waiting time in the queue is 0.2314 minutes and the

expected total time lost waiting in one day is 0.3236 minutes.
The expected number in the queue is 0.0133 patient. The expected number in the system is approximately 2 patients. The expected waiting time in the queue is 0.038 minute and the expected total time lost waiting in one day is 0.0532 minutes. Capacity management barriers contribute to longer length of stay rates for patients within the emergency department (ED) setting due to patients encountering systematic barriers within the patient journey through the system (Armony et al., 2015).

Summary

Considering the solution from the analysis, the system's capacity of 208 patients were studied and the arrival rate is 0.3496 minutes while the service rate is 0.1992 minutes. The arrival being greater than the service rate implies that patients have to wait, though the patients won't wait for long. The probability that the servers are idle is 0.065 which shows that the servers will be approximately 7 % Idle and 93% Busy.
For two servers, the expected number in the queue is 5.8554 patients. The expected number in the system is approximately 8 patients. The expected waiting time in the queue is 17.16 minutes and the expected total time lost waiting in one day is 23.722 minutes.
For three servers, the expected number in the queue is 0.459 patient. The expected number in the system is approximately 2 patients. The expected waiting time in the queue is 1.3129 minutes and the expected total time lost waiting in one day is 1.8356 minutes.
For four servers, the expected number in the queue is 0.0809 patients. The expected number in the system is approximately 2 patients. The expected waiting time in the queue is 0.2314 minutes and the expected total time lost waiting in one day is 0.3236 minutes.
For five servers, the expected number in the queue is 0.0133 patient. The expected number in the system is approximately 2 patients. The expected waiting time in the queue is 0.038 minute and the expected total time lost waiting in one day is 0.0532 minutes.

Conclusion

Hospital waiting lists are a global epidemic affecting almost all medical and surgical facilities, especially in the urban areas. From the summary, we conclude that the usage of three or four servers will help reduce the time patients spend on the queue and also help reduce the cost incurred from waiting. Hence, the advantage of using the multiple server queue is that a slow server does not affect the movement of the queue. With an ageing population and pressures on its healthcare system, Nigeria is no exception. Although many specialities struggle with long wait times, AED appears to be particularly burdened. Mater Miscericordia hospital management is responsible for recruiting and grooming qualified staff through their training school. Training and development policies and procedures must be reviewed in depth for all staff. Healthcare personnel must have appropriate degrees, credentials, state licenses, and the necessary skills to do the job.

Recommendations

We recommend that the management of the hospital should increase the number of servers at the Accident and Emergency Department to three or four so as to reduce the time patients spend in queue before services thereby, minimizing the cost incurred from waiting and also increasing patient's satisfaction with the services rendered. Ahmadpour, Bayramzadeh & Aghaei (2021) supported the recommendation to implement a fast-track triage process. Fast-track systems may also improve patient flow through the ED. A fast-track system diverts patients with non-urgent medical needs into a separate path. Mater Misceriecordie Hospital should consider utilizing its internal strength of nursing school and build an efficient and effective workflow system

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